SHORT TERM SCIENTIFIC MISSION (STSM) SCIENTIFIC REPORT

This report is submitted for approval by the STSM applicant to the STSM coordinator

Action number:
STSM title: Advanced optimization methods for multi-objective robust design
STSM start and end date: 09/12/2018 to 14/12/2018
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PURPOSE OF THE STSM:

The objectives of the STSM are

1. Extend the formulation of the reference model such as to cover correlated facility failures, conditions and/or situations of facility load increase,
2. Extend the applicability of the demand re-routing/protection model to network infrastructure failures
3. Use or even enhance advanced optimization methods including adjustable robust optimization and variable decomposition methods to solve such multi-objective reliability problems. .

The purpose of this mission includes also the elaboration of more detailed description of the failure situations in combination with applicability scenarios this model/these models enable to cover (and to which extend).

DESCRIPTION OF WORK CARRIED OUT DURING THE STSMS

1. The formulation of the capacitated Reliable Facility Location Problem (cRFLP) model used for comparison purposes in this Chapter has been extended to cover in addition to uniform failure probability distributions and unconditional events i) non-uniform failure probability distributions and unconditional failure events and ii) non-uniformin failure probability distributions and conditionally dependent failure events.

   It is to be emphasized that the correlated variant of this problem is not covered by current scientific and technical literature on location theory: only the uncorrelated variant for the cRFLP - the version document in the initial version of this Chapter - and the correlated variant for the uncapacitated RFLP (uRFLP) have been reported.

2. The applicability of the demand re-routing model has also been extended to cover multi-link protection scenario (thus network infrastructures failures) beyond un/correlated facility failures. This scenario extends the applicability of the combined multi-product capacitated Facility Location Routing (cFRLP) model detailed in the current version of the Chapter.

3. The analysis carried out shows that probabilistic models limit the complexity of the non-uniform and the correlated variants of the problem to nonlinear programs (quadratic in the allocation variables) that can be linearized with relatively standard procedures. The situation is different when formulating

the robust counterpart of these problems since the level of complexity of the robustified problem increases its complexity class.

Note: two seminars (1h30 and 2h00) were held during this scientific mission to provide a detailed presentation of the formulations and the results obtained as documented in the initial verison of the chapter.

DESCRIPTION OF THE MAIN RESULTS OBTAINED

The following additional pages to the Chapter describe the main results obtained:

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5 Reliability

In order to enable the dynamic reallocation of demands upon failure of their assigned facility, we take advantage of the re-routing capability of the proposed combined model. For this purpose, we assume that the flow routing decision process is aware of the distribution of the products per facility (and their availability); we assume the existence of a non-failable entity which behaves similarly to a depot that replenishes remaining facilities with data objects that became inaccessible after the failure of their (primary) hosting facility. Hence, upon failure of the facility location to which they are assigned, demands are re-routed to other facilities (not impacted by the failure). This resilience scheme has to be contrasted with one involving demand protection realized in anticipation to any potential facility failure. Such scheme is typically executed without any detailed information (beside pairwise graph distances) about the network design and flow routing decisions.

The demand protection scheme considered is based on the reliability modeling technique introduced in [17] that relies on the levels assignments strategy. This strategy assigns client demands to facilities at multiple levels $0, \ldots, J-1$, which denote the level r at which a facility serves a given client demand. A level$-r$ facility assignment for a client demand will serve this demand if and only if all its assigned facilities at lower levels $0, \ldots, r-1$ have failed. For a two-level scheme, when $r = 0$, we refer to a primary assignment and when $r = 1$ to a backup assignment. Moreover, to capture the probability that all of the J facilities may fail, if a client demand is assigned to exactly J facilities at levels $0, \ldots, r-1$, it shall also be assigned at level r to a non-failable emergency facility (unless this demand is already assigned to an emergency facility at certain level $s < r$). Note that our demand protection scheme involves more elaborated emergency strategies.

5.1 Demand Protection Scheme (upon facility failure)

The properties of the models (detailed in the subsequent sections) underlying the demand protection scheme upon facility failure are summarized in Table 5.

Table 5: Demand Protection: Model Properties

Property	Model 1 (Section 5.1.1)	Model 2 (Section 5.1.2)	Model 3 (Section 5.1.3)
Failure Events	Independent	Independent	Conditionally Dependent
Individual Failure Probability Distribution	Uniform	Non-uniform	Non-uniform
Joint Failure Probability	Individual	Individual	Individual & Conditional
Model complexity	Linear	Nonlinear Quadratic	Nonlinear Quadratic

5.1.1 Uniform Failure Probability - Independent Events

The first demand protection scheme relies on the capacitated variant of the RFLP model introduced by [19]. The cRFLP performs as follows i) a given facility j can only serve a client demand at

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$$\min \sum_{j \in \mathcal{J}} \varphi_j y_j \quad (27)$$

$$+ \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \sum_{r=0}^{J-1} \delta_{ij} a_{ik} x_{ijkr} q^r (1-q) \quad (28)$$

s.t.:

$$\sum_{j \in \mathcal{J}} x_{ijkr} = 1 \qquad i \in \mathcal{I}, k \in \mathcal{K}, r = 0,.., J-1 \quad (29)$$

$$z_{jt} \leq y_j \qquad j \in \mathcal{J}, t \in \mathcal{T} \quad (30)$$

$$x_{ijkr} \leq z_{jt} \qquad i \in \mathcal{I}, j \in \mathcal{J}, k \in \mathcal{K}, t \in \mathcal{T}, t = k[0], r = 0,.., J-1 \quad (31)$$

$$\sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}} \sum_{r=0}^{J-1} a_{ik} \omega_{ijk} x_{ijkr} \leq b_j y_j \qquad j \in \mathcal{J} \quad (32)$$

$$\sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}} \frac{1}{|k|} a_{ik} \leq \sum_{j \in \mathcal{J}} b_j y_j \quad (33)$$

$$\sum_{r=0}^{J-1} x_{ijkr} \leq 1 \qquad i \in \mathcal{I}, j \in \mathcal{J}, k \in \mathcal{K} \quad (34)$$

$$x_{ijkr} \in \{0,1\} \qquad i \in \mathcal{I}, j \in \mathcal{J}, k \in \mathcal{K}, r = 0,.., J-1 \quad (35)$$

$$y_j \in \{0,1\} \qquad j \in \mathcal{J} \quad (36)$$

$$z_{jt} \in \{0,1\} \qquad j \in \mathcal{J}, t \in \mathcal{T} \quad (37)$$

Figure 14: MIP formulation of the cRFLP

single-sourcing property, the allocation variables x_{ijkr} are binary. Hence, in addition to the facility location cost, the objective for a primary-backup demand protection scheme consists of minimizing the routing cost for the primary assignment of each demand together with the routing cost for assigning a backup facility to serve that demand (proportionally to the graph distance δ_{ij} between the corresponding demand point i and the assigned facility j) in case its primary assigned facility fails. The cRFLP generalizes this model to multiple levels $r = 0, 1, \ldots, J-1$.

The MIP formulation of the cRLFP is presented in Fig.14. In the objective function (see (27)), the first term determines the total facility location cost and the second term the expected transportation cost when facility j serves demand point i if its lower-level assigned facilities are all disrupted (occurrence probability q^r) and facility j remains still available (occurrence probability $1-q$). Since in our case, we consider a two-level protection scheme, each demand is assigned to one primary facility at level $r = 0$ and to another (backup) facility at level $r = 1$. Constraints (34) ensure that an installed and opened facility j can only serve a client demand at no more than one level.

5.1.2 Non-uniform Failure Probability - Independent Events

The base (uncapacitated) variant of the RFLP introduced by [17] and extended to its capacitated variant by [19] assumes that each potential facility $j \in \mathcal{J}$ is subject to a uniform failure probability q. That is all facilities have the same probability of failing. To cope with more general situations/scenarios, we have extended the model to non-uniform failure probabilities, i.e., the failure probability q is replaced by a per-facility or a per-site location failure probability q_j such that $0 \leq q_j < 1, \forall j \in \mathcal{J}$. Observe that now not only capacity constraints may increase the transportation/routing cost (when demands have to assigned to farther facilities when the nearer ones don't provide enough capacity) but also the per-facility failure probability since it may be more optimal to assign a client demand to a facility that is farther away but less likely to fail.

Even though the base model covers the case of multiple failures occurring simultaneously and constraints (34) require that a facility can only serve a demand at no more than one level, the demand allocation strategy at each level $r \geq 0$ depends on the occurrence of facility failure knowing that a demand is served at that level instead of relying on the individual facility failure probability or the knowledge that another facility has failed (conditional independent events). Indeed, this variant of the model enables to cover situations where two failure events occurring at facility $j = 1$ and $j = 2$ respectively are conditionally independent -given the occurrence of a disaster event for instance- albeit, given knowledge of whether the disaster event occurs, knowledge of whether facility $j = 1$ fails provides no information on the likelihood of the failure occurrence of facility $j = 2$, and knowledge of whether facility $j = 2$ fails provides no information on the likelihood of the failure occurrence of facility $j = 1$. These conditions determine the common form of conditional independence, where events are not statistically independent but they are conditionally independent.

Consider that the primary single assignment (at level $r = 0$) to facility j of a demand originated by client i depends on the availability of that facility with probability $P_{ijk0} = (1 - q_j)$, where $q_j (0 \leq q_j < 1)$ corresponds to the individual failure probability of facility $j \in \mathcal{J}$. At level $r = 1$, the probability that facility j serves client i at level $r = 1$ is then determined by $P_{ijk1} = (1 - q_j) \sum_{p \in \mathcal{J}} q_p/(1 - q_p) P_{ipk0} x_{ipk0}$, thus $(1 - q_j) q_p$ given that facility p serves client i at level $r = 0$. The simplification included as part of the base model, renders its formulation easy enough to be solved on large instances. The proposed generalization to non-uniform failure probabilities increase the level of realism of the model at the detriment of increasing its complexity. Indeed, for the extended model, the objective function is more involved

$$\sum_{j \in \mathcal{J}} \varphi_j y_j + \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \delta_{ij} a_{ik} \left(P_{ijkr(=0)} x_{ijkr(=0)} + P_{ijkr(=1)} x_{ijkr(=1)} \right), \tag{38}$$

together with

$$P_{ijkr(=0)} = P_{ijk0} = (1 - q_j) \tag{39}$$

$$P_{ijkr(=1)} = P_{ijk1} = (1 - q_j) \sum_{p \in \mathcal{J}} q_p/(1 - q_p) P_{ipk0} x_{ipk0} \tag{40}$$

Thus, the second term of the objective function (38) includes the product $P_{ipkr} x_{ijkr}$ of two binary variables P_{ipkr} and x_{ijkr}. To linearize this product, one introduces an auxiliary variable $\zeta_{ijkr} = x_{ijkr} P_{ipkr}$ together with the following set of constraints i) $\zeta_{ijkr} \leq x_{ijkr}$, ii) $\zeta_{ijkr} \leq P_{ijkr}$, and iii) $\zeta_{ijkr} \geq x_{ijkr} + P_{ijkr} - 1$.

Observe that the formulation of this model for demand protection, which involves binary variables as consequence of the single sourcing property of the model, can be relatively straightforwardly linearized. Though the resulting formulation is polynomial in size, this procedure increases nevertheless both dimensionality (due to the introduction of auxiliary variables) and size of the problem (due to the additional associated constraints) in addition to potentially producing for large-scale instances a significant gap between the IP and the LP relaxation. In general, the single assignment property yields formulations of facility location models whose solving are more computationally demanding than formulations involving fractional assignment variables x. In the latter case however, their product does not allow anymore the application of the above linearization procedures. For products of lower- and upper-bounded continuous variables (in formulations involving bilinear terms), the linearization procedure involves techniques such as McCormick envelopes and piecewise McCormick relaxation, a convex relaxation techniques which can provide a sufficiently tight lower bound.

5.1.3 Non-uniform Failure Probability - Conditionally Dependent Events

Moreover, even with individual non-uniform failure probabilities, facility failures remain independent events and their probability of occurrence unconditional, i.e., simultaneity of events does not translate the potential inter-dependence between facility failure events. Such situations may occur due to the spatial correlations among facility failures/disruptions; in such conditions, neighboring facilities are more likely to fail simultaneously, and clients will find it more costly to reach an available facility that would be able to serve their demands, i.e., offering the requested products k. Temporal correlations between failure events may also occur due to (sometime unknown/unprecedented) ageing effects that would lead to similar remedial actions involving the re-allocation of client demands from their primary assigned facility (at level $r = 0$) to their assigned backup facility (at level $r = 1$). Other situations due to temporally correlated events include maintenance actions that affect a subset of the installed facilities following massive attacks/denial-of-service events. Note that even more complex cascades of event patterns may arise that affect a subset of installed facilities that may not necessarily be anticipated.

To cover the cases where facility failures can be correlated, we have extended the model to joint probability distributions for conditionally dependent failure events, enabling to cover situations where both primary and backup demand assignments are not determined independently. Consider that the primary single assignment (at level $r = 0$) to facility j of a demand originated by client i depends on the availability of that facility with probability $P_{ijkr(=0)} = (1-q_{j0})$, where q_{j0} represents the unconditional individual failure probability of facility $j \in \mathcal{J}$. At level $r = 1$, the probability that facility j serves client i at level $r = 1$ is then determined given that facility p serves client i at level $r = 0$ by $P_{ijkr(=1)} = (1 - q_{j|p})q_{p0}/(1 - q_{p0})P_{ipk0} = (1 - q_{j|p})q_{p0}$ instead of $(1 - q_j)q_p$. The probability $q_{j|p}$ translates the failure probability of facility j given the failure occurrence of facility p with probability q_p. The objective function is then becomes

$$\sum_{j \in \mathcal{J}} \varphi_j y_j + \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \delta_{ij} a_{ik} \left(P_{ijkr(=0)} x_{ijkr(=0)} + P_{ijkr(=1)} x_{ijkr(=1)} \right), \tag{41}$$

$$
\begin{aligned}
\min & \sum_{j\in\mathcal{J}} \varphi_j y_j \\
& + \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} \sum_{k\in\mathcal{K}} \delta_{ij} a_{ik} \left(P_{ijkr(=0)} x_{ijkr(=0)} + P_{ijkr(=1)} x_{ijkr(=1)}\right) \\
\text{s.t.:} & \\
& \sum_{j\in\mathcal{J}} x_{ijkr} = 1 && i\in\mathcal{I}, k\in\mathcal{K}, r=0,..,J-1 \\
& z_{jt} \le y_j && j\in\mathcal{J}, t\in\mathcal{T} \\
& x_{ijkr} \le z_{jt} && i\in\mathcal{I}, j\in\mathcal{J}, k\in\mathcal{K}, t\in\mathcal{T}, t=k[0], r=0,..,J-1 \\
& \sum_{i\in\mathcal{I}} \sum_{k\in\mathcal{K}} \sum_{r=0}^{J-1} a_{ik}\omega_{ijk} x_{ijkr} \le b_j y_j && j\in\mathcal{J} \\
& \sum_{i\in\mathcal{I}} \sum_{k\in\mathcal{K}} \frac{1}{|k|} a_{ik} \le \sum_{j\in\mathcal{J}} b_j y_j \\
& \sum_{r=0}^{J-1} x_{ijkr} \le 1 && i\in\mathcal{I}, j\in\mathcal{J}, k\in\mathcal{K} \\
& x_{ijkr} \in \{0,1\} && i\in\mathcal{I}, j\in\mathcal{J}, k\in\mathcal{K}, r=0,..,J-1 \\
& y_j \in \{0,1\} && j\in\mathcal{J} \\
& z_{jt} \in \{0,1\} && j\in\mathcal{J}, t\in\mathcal{T}
\end{aligned}
$$

Figure 15: MIP formulation of the Extended cRFLP

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together with

$$P_{ijkr(=0)} = P_{ijk0} = (1 - q_{j0}) \quad (42)$$

$$P_{ijkr(=1)} = P_{ijk1} = \sum_{p \in \mathcal{J}} (1 - q_{j|p}) q_{p0} / (1 - q_{p0}) P_{ipkr(=0)} x_{ipkr(=0)} \quad (43)$$

As for the independent case with non-uniform failure probability, the second term of the objective function includes the product $P_{ipkr} x_{ijkr}$ of two binary variables P_{ipkr} and x_{ijkr}. To linearize this product, one relies on the procedure/transformation detailed here above.

Observe also that the introduction of conditional failure probability to account for dependent events does not increase the complexity of the model compared to the one involving unconditional (individual) failure probability for independent events. This observation can be again explained by the use of binary allocation variables x following the single-sourcing property of the demand protection model and the pairing of a unique facility at level $r+1$ and a unique facility at level r to account for the transitional probabilities between adjacent levels. One would use fractional allocation variables, the formulation of the objective would be significantly different. Consider for instance, at level $r = 0$, a subset of more than one facility would serve the demands originated by client i, the transitional probabilities would involve more than one fractional allocation variable (since the demand could be served by more than one facility at each level). For the formulation involving binary allocation variables, this result is counter-intuitive since it is commonly assumed/expected that the injection of conditional failure probabilities as part of the problem significantly increases its complexity.

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FUTURE COLLABORATIONS (if applicable)

The second part of the mission will be dedicated to

1. The solving of the problem outlined in point 2) of the mission statement/purpose with representative execution scenarios.

2. Documentation (dedicated Chapter section) of most important/representative subproblems in addition to the one already documented (in relation to content distribution/centric networks) and solving method (already documented in the current version of the Chapter).

3. If time allows: quantify tradeoff (computational cost/tractability vs. solution properties/quality) between robustified formulation and probabilistic formulation of demand protection schemes.