SHORT TERM SCIENTIFIC MISSION (STSM) SCIENTIFIC REPORT

This report is submitted for approval by the STSM applicant to the STSM coordinator

Action number: 38792
STSM title: Robustness quantification and vulnerability identifications of communication networks
STSM start and end date: 01/10/2017 to 07/10/2017
Grantee name: Xiangrong Wang

PURPOSE OF THE STSM:
(max 200 words)

The object of this STSM is to jointly work on robustness quantifications and vulnerability identifications of real-world communication networks. Three main objectives during the visit to the host institution are:

- Investigate as robustness measures in communication networks, the recently proposed graph metrics, (a) effective graph resistance, (b) the maximum variance in nodal spreader capacity in a graph as the heterogeneity of nodal routing capacity, (c) the Kemeny constant, and (d) the best spreader list (the ranked diagonal elements of the pseudoinverse of the Laplacian of the graph, which we call for brevity the *zeta-vector*).
- Identify the most vulnerable node(s), specified by the zeta-vector, whose sequential removals fragment the networks. Compare the zeta-vector based removal strategy with existing identification strategies, such as betweenness, closeness, the principle eigenvector of the adjacency matrix and the degree vector.
- Discover the underlying mechanism for the different performance of those graph metrics via the failure profiles provided by the UdG simulator.

DESCRIPTION OF WORK CARRIED OUT DURING THE STSMS

(max.500 words)

During the visit in University of Girona from 1 Oct to 7 Oct, the STSM is carried out in the following steps:

- **Interpreting the essence of the robustness metrics**

- (a) Effective graph resistance R_G:
  In analogy to the resistance of resistors in electrical circuits, where the increase of resistance decreases the transportation of electrons in the circuit [1], the effective graph resistance measures the difficulty of flow transportation in networks. A lower effective graph resistance indicates a higher efficiency of flow transportation in networks, and thus a more robust network.
  Effective graph resistance, computed in terms of eigenvalues of the Laplacian, is infinite ($R_G = \infty$) for any disconnected graphs. In the scenario of robustness investigation by removing nodes/links in the networks, the effective graph reistance fails to characterize network robustness once the network is disconnected. We propose two possible solutions: (i) replace 0 with a nonzero weight, e.g., the diameter of the remaining graph, for the removed link that disconnects the network. (ii) In view of the definition $R_G = \sum_{i=1}^{N}\sum_{j=1}^{N} R_{ij}$ of the effective graph resistance and in analogy to the definition of the efficiency[2], we suggest the definition of the effective graph conductance as

$$C_G = \sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{R_{ij}}$$

Which avoids the infinite of effective resistance R_{ij} when nodes i and j are disconnected.

- (b) Maximum variance in nodal spreader capacity and (c) Zeta vector
  Van Mieghem et al. [3] propose the zeta vector $\zeta = (Q_{11}^{\dagger},\ Q_{22}^{\dagger}, \dots,\ Q_{NN}^{\dagger})$, which is the diagonal elements of the pseudo-inverse $Q^{\dagger}$ of the Laplacian matrix of a graph, as the nodal spreader list. For flow (e.g., water, gas, current) that is propotional to the potential difference $v_i - v_j$ of any node pairs i and j, the diagonal element $Q_{ii}^{\dagger} = \frac{1}{N}\sum_{j=1}^{N}(v_i - v_j)$ quantifies the average potential difference of a node i to all the nodes in the network. The node with a minimum value of $Q_{ii}^{\dagger}$ is, therefore, regarded as the best spreader node.

  The maximum variance of the zeta vector characterizes the heterogeneity of the nodal speader capacity.
- (d) Kemeny constant:
  For a random walk with the transition probability $P = \Delta^{-1}A$, where Δ^{-1} is the diagonal matrix of nodal degree and A is the adjacency matrix of a graph, Kemeny constant equals to

$$K(\Delta^{-1}A) = \sum_{j=1}^{N}\pi_j m_{ij}$$

  Where π_j is the prabobility of a random walk visiting node j in the steady state and m_{ij} is the hitting time from node i to node j. Kemeny constant characterizes in steady-state the average hitting time of a node to all its neighbours. Kemeny constant can be computed via the pseudo-inverse $Q^{\dagger}$ of the Laplacian [4]

$$K(\Delta^{-1}A) = \zeta^T d - \frac{d^T Q^{\dagger} d}{2L}$$

- **Integrate there robustness measures in the UdG simulator.**

- **A global robustness list, based on the zeta vector, is designed to identify the vulnerable nodes and is integrated in the UdG simulators.** The removals of the identified nodes highly impact the robustness curve of the tested real-world networks.

- **Assess the robustness measures and identification algorithm in real-world networks**

DESCRIPTION OF THE MAIN RESULTS OBTAINED
(max.500 words)

Robustness evaluation is performed by sequentically removing nodes and measuring the impact of these removals. The robustness investigation, therefore, boils down to the design of strategies for vulnerability identification (or nodal removal), and robustness measurement. To demonstrate the applicability of the proposed robustness measures and identification algorithm in real-world networks, we perform simulations on real-world networks via the UdG simulator.

Figure 1 shows visualization of network *Abilene* under 30% of nodal removal where the removed nodes, highlighted in red, are identified based on tha value of the zeta vector. Moreover, the UdG simulator [5] provides the visualization of the dynamical failure profiles. The heatmap, as shown in Figure 2, illustrates the decrease of the robustness value with the increase of removed nodes in network *Deltacom*. Figure 3 shows the robustness curve as a function of the sequencial removals according to the zeta vector.

Figure 1: *Abilene* under 30% of nodal removal

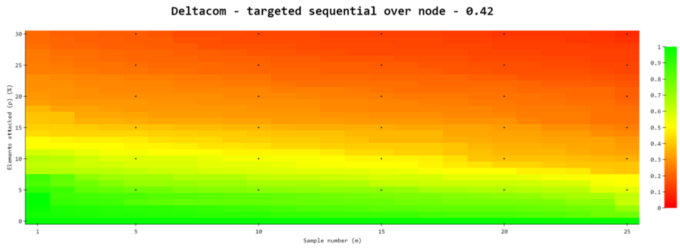

Figure 2: Heatmap for the decrease of the robustness

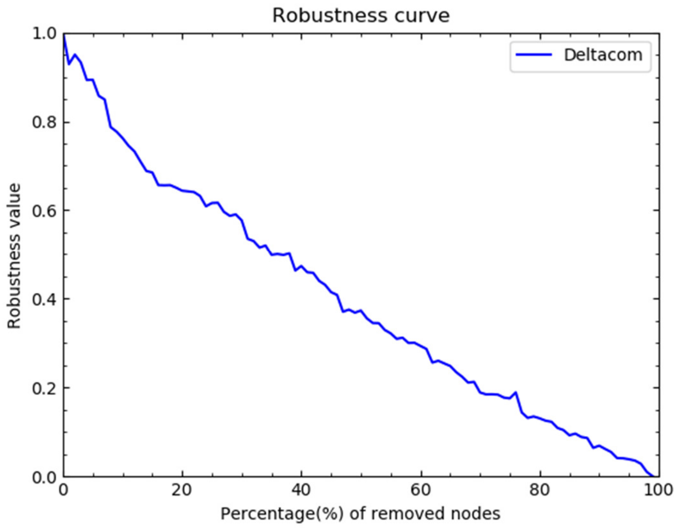

Figure 3: Robustness curve as a function of nodal removals

FUTURE COLLABORATIONS (if applicable)

The UdG simulator integrates critical infrastructures with geographical coordinates. In the scenario of limited budget, it is economically efficient to interconnection different infrastructures according to the geographical locations. The future collaborations aims at modelling the interactions between real-world infrastructures incorporating the geographical coordinates.

References:

1. Klein, Douglas J., and Milan Randić, 1993, "Resistance distance." Journal of mathematical chemistry 12.1: 81-95.
2. Latora, Vito, and Massimo Marchiori, 2007, "A measure of centrality based on network efficiency." New Journal of Physics 9.6:188.
3. Van Mieghem, P., K. Devriendt and H. Cetinay, 2017, "Pseudo-inverse of the Laplacian and best spreader node in a network", Physical Review E, vol. 96, No. 3, p 032311.
4. Wang, X. , J. L. A. Dubbeldam and P. Van Mieghem, 2017, "Kemeny's constant and the effective graph resistance", Linear Algebra and its Applications, vol. 535, pp 231-244.
5. UdG simulator: http://songohan.udg.edu